EXTRA PRACTICE PROBLEMS - FINAL EXAM

1. Chapter 1.1-1.4, 3.1-3.3, 4.1-4.4, 5.1-5.3

(1) Sales figures show that your company sold 1960 pen sets seach week when they were priced at \$1 per pen set and 1800 pen sets each week when they were priced at \$5 per pen set. What is the linear demand function for your pen sets?

Answer: q = -40p + 2000

(2) System of linear equations

(3) The following table show the average price of a two-bedroom apartment in downtown Toronto from 2004 to 2014 in million dollars. ($t = 0$ represents 2004).

Year (t)	0	2	4	6	8	10
Price (p)	0.38	0.40	0.60	0.95	1.20	1.60

(a) Find the linear regression line and correlation coefficient r, with all coefficients rounded to two decimal places. Ploit the regression line and the given points.

Answer: $p = 0.13t + 0.22$; $r \approx 0.97$

(b) What does the value of r suggest about the relationship between t and p.

(c) Jim's accountant informs you that he will be able to purchase an apartment under 1.8 million. Based on your answer in part a) will he be able to afford the apartment in 2016.

Answer: Based on answer in part a), the apartment will cost 1.78 million in 2016 so he can afford the apartment.

(4) The demand and supply functions for college newspaper are, $q = -10000p + 2000$ and $q = 4000p + 600$ respectively, where p is the price in dollars. At what price should the newspapers be sold so that there is neither a surplus nor a shortage of papers?

Answer: Equilibrium price p = \$ 0.10

(5) Use Gauss-Jordan row reduction to solve the given system of equations:

$$x + y + 2z = -1$$

$$2x + 2y + 2z = 2$$

$$\frac{3}{5}x + \frac{3}{5}y + \frac{3}{5}z = \frac{2}{5}$$

Answer:

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & -1 \end{array}\right)$$

This is an inconsistent system. No solution.

(6) Chapter 3.3, #16

(7) Chapter 3.3 #17.

(8) Let

$$A = \begin{bmatrix} 0 & -1 \\ 3 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$$

Find:

(a) $A + B$

(b) $A^{-1} \cdot B^{-1}$

(c) $A^T + B^T$.

(9) Chapter 4.3, Question 53

(10) Reduce the payoff matrix

$$\begin{array}{c} \\ p \\ q \\ r \\ s \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} 2 & -4 & 9 \\ 1 & 1 & 0 \\ -1 & -2 & -3 \\ 1 & 1 & -1 \end{bmatrix} \end{array}$$

by dominance.

Answer: $\begin{array}{c} \\ p \\ q \end{array} \begin{array}{c} \begin{array}{cc} b & c \end{array} \\ \begin{bmatrix} -4 & 9 \\ 1 & 0 \end{bmatrix} \end{array}$

(11) Determine whether the game is strictly determined. If it is, give the optimal pure strategies and value of the game:

$$\begin{array}{c} \\ p \\ q \\ r \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} -7 & -3 & 6 \\ 5 & -1 & -4 \\ 1 & 0 & 2 \end{bmatrix} \end{array}$$

Answer:

$$\begin{array}{c} \\ p \\ q \\ r \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} \textcircled{-7} & -3 & 6 \\ 5 & -1 & \textcircled{-4} \\ 1 & \textcircled{0} & 2 \end{bmatrix} \end{array} \qquad \begin{array}{c} \\ p \\ q \\ r \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} -7 & -3 & \boxed{6} \\ \boxed{5} & -1 & -4 \\ 1 & \boxed{0} & 2 \end{bmatrix} \end{array}$$

Therefore we see that 0 is both a row minima and a column maxima, and therefore is a saddle point. Therefore the optimal strategy for the row player is u and the optimal strategy for column player is b and the expected value of the game is 0.

(12) Chapter 4.4, #21
(13) Chapter 4.4, #27
(14) Chapter 5.2, Example 4.
(15) Chapter 5.2, #21.
(16) Chapter 5.3, #27.
(17) Chapter 5.3 #39.

2. Calculus

I strongly recommend working through **Homework** 6, 7, 8, 9, 10 as well the questions posted during the start of the semester for practice with integration and differentiation, if you have not already.
Also work through the **differential equations handout** if you have not yet. Those questions are not included in this review guide. On top of those, I want you to work on the problems listed below.

(1) Use the defintion of derivative to evaluate derivative of the function $f(x) = \dfrac{1}{2x+3}$.
(2) Chapter 10.5 #59 – #64
(3) Chapter 10.5 #89.
(4) Chapter 10.6, Example 5 (Pg. 765)
(5) Chapter 11.2, #11.
(6) Find the equation of the tangent line to the curve $y = e^x \log_2(x)$ at point $(1, 0)$.
Answer: $\boxed{y = \dfrac{e}{\ln 2}(x-1)}$
(7) Chapter 12.2 #23.
(8) Chapter 12.2 #33.
(9) Chapter 12.2 #59.

(10) Chapter 13.1 #47.

(11) Chapter 13.2 #29.

(12) Chapter 13.3, Example 4.

(13) Chapter 13.4, Example 1.

(14) Chapter 13.4 #81.

(15) Chapter 14.6 #19.

(16) Chapter 14.6 #21.

(17) Chapter 15.1 #75.

(18) Chapter 15.1 #97.

(19) Chapter 15.2 #47.

(20) Locate and classify all critical points of the function $f(x, y) = e^{xy}$. (Note that $e^a > 0$ for any real number a).

Answer: $\boxed{\text{Saddle point at } (0,0)}$.

(21) Chapter 15.4 #23

(22) Chapter 15.4 #19.

(23) Chapter 15.4 #39.